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Dynamic analysis of composite beam with piezoelectric layers under thermo-mechanical load

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Abstract. In this paper, the control of composite beam vibrations with sensor and actuator connected layers is considered with consideration of the effect of thermal environment. The coupling relation between electrical field and mechanical deformation with uncoupled thermal impact are used. The mathematical model of shear deformation (Timoshenko's theory) has been applied and basic equations for piezoelectric sensors and actuators have been proposed. The equation of motion for the beam structure is obtained by the Hamilton principle and analyzed by finite element method. The control algorithm is based on proportional velocity control. Hence, the purpose of this article is to investigate the direct and inverse effects of piezoelectric on control of simply supported beam vibration under uniform temperature.

1. Introduction

Temperature variation is one of the effective or sometimes a major factor in the destruction of engineering structures. The structure members, for example, in reactor shells, spatial structures, turbines and other components of the mechanism, are prone to breakage due to buckling, deflection with high ranges or additional stresses due to thermal loads or mechanical-thermal composite loads. The development of composite materials with piezoelectric components has great potential in terms of safety and structural control [1-4]. Initially, piezoelectric phenomenon is used as a collection of mechanical energy from vibrations [5]. Due to their direct and reciprocal effects, these new materials, with their inherent and adaptive properties, can demonstrate better mechanical and thermal properties and, in fact, eliminate problems and problems caused by thermal loads or mechanical thermal loads. Given that these materials are lightweight, compact, simple, solid and also small in size, a large number of them can be used in the structure without significantly increasing the structure's mass. Piezoelectricity was first discovered by the Curie Brothers in 1880. They found that by applying pressure to a quartz crystal, an electric field is produced in the crystal [6]. The piezoelectricity theory, along with the thermal field, is called the theory of thermo-piezoelectric and has been worked out in limited articles [7, 8]. The governing equations for thermo-piezoelectric were first extracted by Mindlin [9]. Using the general theory of Hamilton, Nowacki obtained the general principle of thermo-piezoelectric [10]. Tauchert has been instrumental in controlling the force vibrations caused by very rapid thermal or mechanical arousal [11]. Saravanos and Lee have performed thermo-piezoelectric analysis of composite beam using layerwise theory [12]. Sunar and Rao have done research on the use of piezoelectric materials in places where temperature changes are significant [13].



Here, the two sides pin beam with piezoelectric layers attached to the upper and lower surfaces is considered, and the effect of piezoelectric materials on the behavior of the beam in a dynamical state is investigated. A finite element model using the Gelarkin method has been used based on the Timoshenko beam theory to analyze the beam performance, under the influence of distributed force on the upper beam and thermal load, using the Newmark method.

2. Extracting governing equations

The displacement field components in Timoshenko's beam are considered in the form below [14]:

$$u(x, z, t) = z\varphi_x(x, t) \quad (1)$$

$$w(x, z, t) = w_0(x, t) \quad (2)$$

Thus, the axial strain components and the shear strain are

$$\varepsilon_{xx} = z \frac{\partial \varphi_x}{\partial x} \quad (3)$$

$$\varepsilon_{xz} = \varphi_x + \frac{\partial w_0}{\partial x} \quad (4)$$

2.1. Thermo-piezoelectric equations

$$\{\sigma\} = [c]\{\varepsilon\} - [e]^T \{E\} - [\lambda]\theta \quad (5)$$

$$\{D\} = [e]\{\varepsilon\} + [\varepsilon_p]\{E\} - [P]\theta \quad (6)$$

The equation (5) is called the inverted thermo-piezoelectric effect, and equation (6) is called direct thermos-piezoelectric effect [12, 13]. It is assumed that the layer of the actuator with thickness h_p , voltage (V^e) is applied only in the direction of the thickness of the actuator layer. As a result, the electric field vector is defined as follows [13]:

$$\{E\} = \begin{bmatrix} 0 & 0 & \frac{1}{h_p} \end{bmatrix}^T V^e \quad (7)$$

2.2. Governing equations of beam

To derive the governing equations for moving the beam, we use the principle of virtual displacement defined as:

$$\delta \int_{t_0}^{t_1} (T - U + W) dt = 0 \quad (8)$$

where δT , δU and δW are virtual kinetic energy, virtual strain energy and virtual work respectively, which are done by external forces.

Using the displacement field of the beam (Eqs. (1) and (2)) and strain equations (Eqs. (3) and (4)), and using the dynamic model of the principle of virtual displacement, It can be obtained the governing equations of motion beam, taking into account Timoshenko's theory, according to the following:

$$\begin{aligned}\frac{\partial Q_x}{\partial x} + q &= I_0 \frac{\partial^2 w_0}{\partial t^2} \\ \frac{\partial M_{xx}}{\partial x} - Q_x &= I_2 \frac{\partial^2 \varphi_x}{\partial t^2}\end{aligned}\quad (9)$$

where

$$M_{xx} = b \int_{-h/2}^{h/2} \sigma_{xx} z dz \quad (10)$$

$$Q_x = b k_s \int_{-h/2}^{h/2} \sigma_{xz} dz \quad (11)$$

$$\begin{Bmatrix} I_0 \\ I_2 \end{Bmatrix} = b \int_{-h/2}^{h/2} \rho \begin{Bmatrix} 1 \\ z^2 \end{Bmatrix} dz \quad (12)$$

Parameters M_{xx} , Q_x , (I_0, I_2) , b , ρ and (k_s) are respectively result moment, shear force, mass moments inertia, width of beam, density and shear correction coefficients. Thus, The shear correction factor is considered $k_s = 5/6$ [14].

2.3. One dimensional thermal loading

A double-sided pin beam with a thickness of h , where boundary conditions ($z = -h/2$ and $z = h/2$) are constant. So that θ_b and θ_t are represented respectively temperature of the below and top of the beam. Thermal energy is not produced inside the beam.

The temperature distribution in thermos-piezoelectric composite beam is

$$\Delta\theta = \frac{\theta_b + \theta_t}{2} + z \frac{\theta_b - \theta_t}{h} \quad (13)$$

Thus, thermal stress is obtained from equation (13). The mechanical stress is acquired with substituting equations ((3) and (4)) into equation (5). Substituting the thermal and mechanical stresses into equations ((10) , (11)), and then substituting along with equation (12) into equation (9), the governing equations motion of thermo-piezoelectric of beam will be obtained.

2.4. Sensor and actuator equations

2.4.1. Sensor equation. A layer is used as a sensor layer in a distributed thermo-piezoelectric composite system. Based on the theory of direct piezoelectric effects in a distributed system, it can obtain the effects of the strain of the beam and the field of heat. To avoid error, the distributed thermo-piezoelectric sensor layer has a very small thickness compared to the total thickness of the beam.

Electric charge using Gaussian theory is as follows [15]:

$$Q_s^k = \frac{1}{2} \left[\int_{z=z_{k+1}} D_z dz + \int_{z=z_k} D_z dz \right] \quad (14)$$

The relation of electric field with the difference of electrical potential is as follows [16, 17]:

$$E = \frac{V_s}{h_{ps}} \quad (15)$$

where h_{ps} is thickness of sensor layer.

By putting the equation (6) in equation (14), the electric field is obtained and then by placing in the equation (15), the difference in the electrical potential of the sensor layer is obtained.

2.4.2. Actuator equation. In the direct proportional control system, the output voltage equation of the sensor with the applied voltage of the operator is as follows:

$$V_a = GV_s \quad (16)$$

By placing the equation (16) into equation (15), the electric field of actuator is obtained. Then, the result put into equation (6).

3. Finite element

Here, by finite element method, the Timoshenko beam equations with the aid of the Galerkin method, are obtained mass, hardness and force matrices.

3.1. Integral form

By means of applying the Galerkin method which equation (9) are multiply by weight functions (M_1, M_2) . Then with integration relative to a length of an element is

$$\begin{aligned} \int_0^L \left[\frac{\partial Q_x}{\partial x} + q - I_0 \frac{\partial^2 w_0}{\partial t^2} \right] (-M_1) dx &= 0 \\ \int_0^L \left[\frac{\partial M_{xx}}{\partial x} - Q_x - I_2 \frac{\partial^2 \varphi_x}{\partial t^2} \right] (-M_2) dx &= 0 \end{aligned} \quad (17)$$

3.2. Model of finite element

The equation (17) are the integral form of the equations of the beam. Then with approximation φ_x and w_0 according to the integral form of φ_x and w_0 are relative to x. Therefore, these approximation are approximated throughout the length of element:

$$\begin{aligned} \varphi_x &\approx \varphi_x^e = \sum_{j=1}^m \varphi_j^e n_j^e \\ w_0 &\approx w_0^e = \sum_{j=1}^n w_j^e m_j^e \end{aligned} \quad (18)$$

where, φ_j^e and w_j^e are at j node. The weight functions also are defined as follows:

$$\begin{aligned} M_1 &= m_j \\ M_2 &= n_j \end{aligned} \quad (19)$$

m_j^e and n_j^e are obtained by choosing the type of element and its form that is considered linear. Replace the approximation equation (equation (19)) into the integral forms (equation (17)). Thus, the results are obtained in form of mass, stiffness and force matrices:

$$\begin{aligned}
K_{ij}^{11} &= \int_0^L GA \frac{dm_i}{dx} \frac{dm_j}{dx} dx & K_{ij}^{12} &= \int_0^L GA \frac{dm_i}{dx} n_j dx \\
K_{ij}^{21} &= \int_0^L GA \frac{dm_j}{dx} n_i dx & K_{ij}^{22} &= \int_0^L \left(EI \frac{dn_i}{dx} \frac{dn_j}{dx} + GAn_i n_j \right) dx \\
m_{ij}^{11} &= \int_0^L I_0 m_i m_j dx & m_{ij}^{22} &= \int_0^L I_2 n_i n_j dx \\
F_i^1 &= Q_{2i-1} + \int_0^L m_i q_m dx & F_i^2 &= Q_{2i}
\end{aligned} \tag{20}$$

3.3. Shape function

The functions w and φ are considered first-order linear:

$$\begin{aligned}
w &= m_1 w_1 + m_2 w_2, & \varphi &= n_1 \varphi_1 + n_2 \varphi_2 \\
m_1 &= n_1 = 1 - \frac{x}{L_e}, & m_2 &= n_2 = \frac{x}{L_e}
\end{aligned} \tag{21}$$

By placing equation (21) in equation (20), the matrix form of the equation (20) are obtained. As a result, the general form of the finite element of uncoupled dynamical thermo-piezoelectric equations in closed-circuit mode in the direct proportional control are as follows:

$$\begin{aligned}
(Mx)_1 + (Kx)_1 &= (F_{mec})_1 + (F_{th})_1 + (F_{boundary})_1 \\
(Mx)_2 + (Kx)_2 &= (F_p)_2 + (F_{th})_2 + (F_{boundary})_2
\end{aligned} \tag{22}$$

4. Results and discussion

The double-headed joint (simply supported) is made of graphite-epoxy which a combination of two symmetrical angular layers (angle-ply) $[45/-45]_s$ with a total thickness of beam (35 mm), a width of beam (50 mm) and a length of composite beam (350 mm) in initial temperature 298.15° K. Piezo-electric layers on the sides of the beam are made of piezoceramic (PZT) with thickness of 0.8 mm. The properties of the material are in Table 1.

Table 1. Material properties [12].

Property	Graphite/epoxy	Piezoceramic (PZT) APC 840
Elastic stiffness, E_{11} (Gpa)	39	68
Elastic stiffness, E_{22} (Gpa)	8.6	68
Density (Kg/m^3)	2100	7600
Shear modulus G_{12} (Gpa)	3.8	26.2
Major poisson's ratio ν_{12}	0.28	0.3
Major poisson's ratio ν_{12}	0.06	0.3
Piezo-electric Constant, e_{31} (m/V)	0	-125×10^{-12}
Therma Expansion Coefficient λ_{11} ($1/^\circ\text{C}$)	7×10^{-6}	3.8×10^{-6}
Pyroelectric constant, P_3 ($\text{C/m}^2 \text{ } ^\circ\text{C}$)	0	-0.25×10^{-3}
Electric permittivity, ε_{33} (N/V^2)	0	11.06×10^{-9}

4.1. Dynamic analysis

In dynamic analysis, a uniform impact force ($F = 400 \exp(-1000t)$), applied on the upper surface of the beam is used to examine the vibration of the beam. To solve, we use the Newark method with linear coefficients $\beta = 0.25, \gamma = 0.5$. The upper piezoelectric layer and the lower layer are considered as a sensor and actuator respectively. The decrease in the range of vibrations, depends on the feedback control gain.

The following graph shows the thermal and mechanical distribution effect on the vibrating system. As the coefficient of damping (gain) increases, the damping force increases and the vibration velocity decreases. Heat term acts as an external force (temperature on top and bottom surface respectively are ($\theta_t = 373 \text{ K}$, $\theta_b = 298.15 \text{ K}$), it increases the range of vibrations. In Figure 1 free vibration of the middle of simply supported beam with thermal force effect and without damping.

The boundary condition of simply supported beam as

$$x = 0, a \quad ; \quad w_0 = w = \varphi_x = M_{xx} = 0$$

The cross section of side boundary are considered to be insulated thermally.

In Figure 2, with the application of feed back control, the electrical gain is increased, with regard to the effect of temperature rise, the amplitude of the vibration of the composite beam is reduced earlier and more.

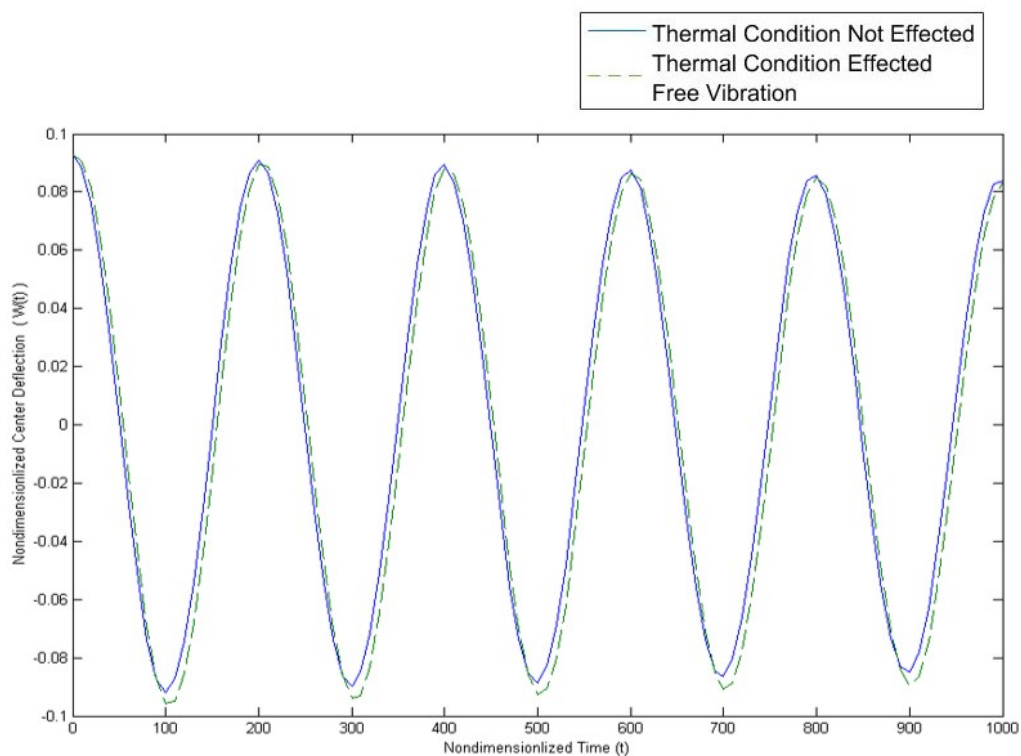


Figure 1. The vibrations in the middle of the beam, taking into account the effect of heat, regardless of the distributed effect of the sensor layer and piezoelectric actuator layer.

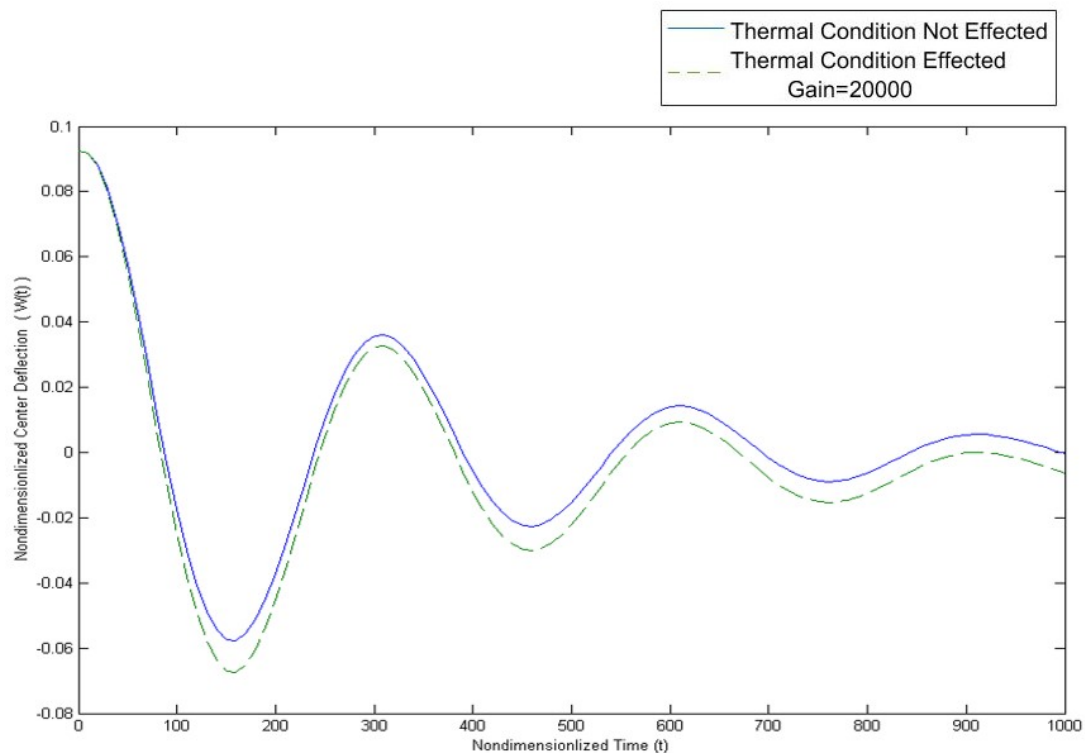


Figure 2. Damping the vibrations in the middle of the beam due to the direct distributed proportional control effect in terms of time, taking into account the effect of temperature rise.

5. Conclusions

Based on the Timoshenko beam model and the principle of virtual work, the governing equations were extracted. For a linear thermal distribution, the deflection of the beam and the effect of the voltage were checked for a decrease. Also, in dynamic analysis, a simple feedback control algorithm called Direct proportional Controlling, which couples the direct and reverse effects of piezoelectric couplings, and the effect of increasing the gain rate on vibration reduction, was investigated. A special thermal distribution that dynamically increases the amplitude of the vibration. It can be concluded that piezoelectric materials have a relatively high efficiency for controlling the shape of structures in environments that are characterized by significant temperature variations.

References

- [1] Raghavan A and Cesnik C E 2005 *Smart Materials and Structures* **14** 1448
- [2] Saravanas D A and Heyliger P R 1995 *Journal of Intelligent Material Systems and Structures* **6** 350-63
- [3] Mancini S, Tumino G and Gaudenzi P 2006 *Journal of Intelligent Material Systems and Structures* **17** 577-85
- [4] Han B, Wang Y, Dong S, Zhang L, Ding S and Yu X 2015 *Journal of Intelligent Material Systems and Structures* **26** 1303-45
- [5] Bowen C, Kim H, Weaver P and Dunn S 2014 *Energy & Environmental Science* **7** 25-44
- [6] Qin Q 2012 *Advanced Mechanics of Piezoelectricity* (Springer Science & Business Media)
- [7] Aouadi M 2006 *International Journal of Solids and Structures* **43** 6347-58
- [8] Chattopadhyay A, Li J and Gu H 1999 *AIAA Journal* **37** 1633-38
- [9] Mindlin R 1974 *International Journal of Solids and Structures* **10** 625-37
- [10] Nowacki W 1985 *Mathematical models of phenomenological piezoelectricity* vol. 1 (Banach Center Publications) p 593-607
- [11] Tauchert T, Adali S and Verijenko V 2001 *4th Int. Cong. on Thermal Stresses* (Cracow) p 8-11
- [12] Lee H J and Saravanas D D 1996 *AIAA Journal* **34** 1231-37

- [13] Sunar M and Rao S S 1997 *AIAA Journal* **35** 534-9
- [14] Wang C, Reddy J N and Lee K 2000 *Shear deformable beams and plates: Relationships with classical solutions* (Elsevier)
- [15] Lam K, Peng X, Liu G and Reddy J 1997 *Smart Materials and Structures* **6** 583
- [16] Shegokar N L and Lal A 2014 *Meccanica*, **49** 1039-68
- [17] Liew K, Yang J and Kitipornchai S 2003 *International Journal of Solids and Structures* **40** 3869-92